RISK-BALANCED TERRITORY DESIGN OPTIMIZATION FOR A MICRO FINANCE INSTITUTION

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ABSTRACT. Micro finance institutions (MFIs) play an important role in emerging economies as part of programs that aim to reduce income inequality and poverty. A territory design that balances the risk of branches is important for the profitability and long-term sustainability of a MFI. In order to address such particular business needs, this paper proposes a novel risk-balanced territory planning model for a MFI. The proposed mixed integer programming model lets the MFI choose the location of the branches to be designated as territory centers and allocate the customers to these centers with respect to planning criteria such as the total workload, monetary amount of loans and profit allocation while balancing the territory risk. This model is solved using a branch and cut based hybrid-heuristic framework. We discuss the impact of the risk balancing and merits of the proposed model.

1. Introduction. Micro finance institutions (MFIs) are important in emerging market economies as part of programs that aim to reduce income inequality and poverty [21]. A risk-balanced territory design is integral to their short and long-term success. This paper presents a real-world multi-criteria territory design problem for a MFI. We utilize a mixed integer programming model that helps the MFI to select the location of the branches to be designated as territory centers and allocate the customer base to these centers with respect to specific planning criteria and risk balancing. The main contributions of this paper are two-fold. Firstly, this is one of the first territory designs proposed for a MFI. In addition, to the best of our knowledge, the proposed mixed integer programming model is the first territory design model in which risk balancing with respect to activity measures is incorporated. A risk-balanced territory design can result in a number of advantages for the network despite the larger total distance travelled. We discuss the impact of the risk balancing and merits of the proposed model.

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1.1. Micro Finance Institutions. Micro finance institutions consist of a collection of banking practices with the objective of making credit and other financial instruments accessible to underprivileged and poor populations. This financial alternative to regular banking has emerged as an important solution to scarce capital for those living in poverty, especially in countries with developing economies [8]. It started with experiments in low-income countries, such as Bangladesh [4]. These experiments intended to challenge the well-established ways of thinking about markets and social policy in low-income communities. Micro finance institutions provide small loans without collateral to customers who would ordinarily be avoided by commercial banks because they represent high risk and low profit. Micro finance presents opportunities for expanding markets, diminishing poverty, and incentivizing social change.

A territory design that promotes an efficient operation is imperative in order for micro finance to be sustainable over the long term. This makes the MFIs resource allocation decisions paramount. The flow of capital and the distribution of services within the network are integral parts of micro finance institutions. As a major difference from the regular banking practice, MFIs generally do not operate or own the branches, they only oversee the credit operations. These branches operate within neighborhood businesses (e.g., gas stations and grocery markets) which enable the MFI to reduce the building and operating costs. The prospective customers, within a given region, generally are assigned to a particular branch, also referred to as territory centers. Each territory center should be chosen in such a way to minimize its distance from groups of customers. Figure 1 provides an illustration of branches and customers for the micro finance institution.

Risk assessment and balancing are crucial matters in micro finance networks. Micro finance institutions must deal with risks that standard banks are not designed to handle. Indeed, economic theory discourages the practice of lending money to low-income households that lack collateral to secure the loans. For this reason, banks have historically avoided serving people in poverty. The risk of lending money to low-income households involves adverse selection, moral hazard, and transaction costs. These sources of risk are associated with information asymmetries, which may exist in the form of faulty screening of projects or ineffective management of small loan disbursements [4]. Reducing information asymmetry by group monitoring, schemes of staggered and progressive lending, micro finance institutions are able to decrease these risks [5]. In addition to risks related to information asymmetries, there are other business risks for a micro finance network. The MFI aims to establish a mutually rewarding relationship with the branches, so that branches remain to be part of the system and the network remains sustainable in the long term. Therefore, risk balancing among branches is important. The MFI prefers to have a network that includes diverse branch types with balanced risk to facilitate the operations.

Despite the importance of territory design and balanced risk, most of the decision-making models within MFI literature omit those topics while focusing on measuring efficiency. For example, Data Envelopment Analysis has been used extensively with different criteria to measure the efficiency of MFIs, see the review by Gutierrez-Nieto et al. [16]. Abdul Hadi and Kamaluddin [1] propose a social collateral model that can help MFI distribute the microfinance loan to the borrowers effectively. Agent-based mapping has been proposed to measure MFI financial risk in a dynamic network setup for borrowers [22], albeit without any consideration of the territories.
The literature of decision models for MFIs, particularly the ones dealing with territory design, is very limited. Bartual Sanfeliu et al. [6] use a weighted multi-criteria goal programming model that aims to maximize the performance of the MFI, without considering territory design. In fact, territory design models are relatively scarce even in the general banking literature [31]. Wang et al. [48] develop budget-constrained location models with the objective of satisfying customer demand. Monterio and Fontes [32] propose a local search heuristic for the location of bank branches. The proposed territory design in this paper is one of the first territory design models for MFIs. To the best of our knowledge, the only exception is the simpler model of López et al. [24]. Their emphasis is the proposal of a novel hybrid heuristic based solution approach for large scale mixed integer programs. They perform comprehensive statistical analysis that examines the interplay of their heuristics and focus on the computational insights. Their model does not consider the proper weight transformations among objective function terms. This limits their work to sole consideration of the distances in their location model, but not the other terms including risk balancing, as in our work. In doing so, they refer
2. Problem Description. The MFI of interest in this paper serves more than two million low-income people in Mexico, a Latin American emerging market. The main objective of this MFI is to remain profitable by managing the associated risks while keeping the customer and network satisfaction at high levels. Its business model differs from that of regular banks. The MFI branches are fully contained within other established businesses. Each branch is located inside, or adjacent to, businesses that attract a large volume of customers and have convenient locations. There are five major branch types: branches within grocery stores, convenience/meat markets, drug stores, gas stations and fast-food restaurants. The MFI has aggregated the existing and potential customers into groups which are referred to as basic units (BUs). Each particular branch is assigned to serve one or more of these basic units. The combination of a branch and the group of customers (BUs) served by that particular branch is referred to as a territory, and that respective branch is designated as the territory center. The MFI designates a pre-determined number of territories to facilitate operations. In order to ensure service delivery to diverse segments of customers, representation of all types of branches is preferred in the territory design. There are pre-specified lower and upper bounds for the number of branches of each type.

A particular business need of this MFI is to have a tight control on the allocation of the basic units to the territory centers. Customers can only get services from their corresponding territory center, which is decided by the MFI. Given this restriction on customers, the territory design needs to meet the customer needs. The MFI seeks customer satisfaction by ensuring that the travelling distance for customers assigned to a particular territory center is as small as possible. Therefore, the objective of the MFI is to construct a territory design so that the total travelling distance of the customers to their assigned territory center is minimized.

In addition, the territory design needs to satisfy the requirements of planning criteria. These so-called activity measures are the total required workload for each set of customers (BU), the total dollar amount of loans for each BU and the total profit due to accrued interests for BU. The MFI and each branch collectively choose target values and tolerance levels for these activity measures. These levels help allocate an activity amount for each branch that it can efficiently handle.

The MFI prefers a design that is similar to the existing Basic Unit-Territory Center assignments. Some geographical considerations are also necessary. For instance, there are some BUs which need to be assigned to different territories due to organizational preferences or geographical obstacles like rivers or mountains. The MFI provided the data that include the distances between the locations of branches to the technical report version of this manuscript as the more complete model. Our manuscript investigates and discusses the merits of risk balancing within a territory design model for a MFI.

This paper is structured as follows. Section 2 defines the real-world territory design problem with an emphasis on the particular business needs, and the definition of risk and risk balancing from the MFI perspective. Section 3 provides a brief overview of the territory design optimization literature and presents the modeling framework, whereas Section 4 introduces the solution framework and utilized heuristics. An analysis of the potential merits of the model is discussed in Section 5. The paper concludes with a discussion of the model limitations, the relevance of risk-balanced territory design for similar problems and potential extensions.
and sets of customers, the variance and total amount of profit, the total workload and total dollar amount of loans. The number of requested territory centers are specified as 500, to serve the needs of given 10,000 basic units.

Although this manuscript is motivated by this Mexican MFI, it can be applied to most of the Latin American MFIs, since they have similar characteristics. It can be argued that differences in the branch types, customer aggregation, number of territories and branches, bound requirements, control on customer allocation, organizational preferences on activity and risk measures can exist with respect to the needs of any particular MFI, especially for the MFIs in Southeast Asia and Africa. However, the proposed model provides a general framework for all MFIs, and can be modified with respect to the potentially unique needs of other MFIs.

2.1. Risk and Risk Balancing for the MFI. Risk is generally defined as the variation in the distribution of the possible outcomes [26]. Heckmann et al. [18] conduct a critical review on supply chain risk, and argue that there is a lack of a universally accepted definition of risk. It is mostly related to the flow and potential disruptions in the network. Deviation-based measures such as variance, absolute deviation [25] and downside risk measures such as Value at Risk [23] are utilized to quantify risk. In addition, some approach or problem specific functions are used to measure risk [33]. In this manuscript, we use a problem specific variance-based function, which is referred to as excess profit variance.

For any commercial network, risk consideration of different aspects of the business model is crucial in addition to the well studied financial risk. For instance, differences in asset values are shown to result in network failures [13]. An et al. [3] explore the effects of aggregation on risk for farmers within an agricultural network, and find a variety of benefits including cost reduction, increased yield and decreased uncertainty. Sodhi and Tang [44] investigate the impact of supply chain design in developing countries, and discuss the research opportunities for supply chain microfinance. In particular, they argue that existing literature in the area of sales territory design can be extended by involving characteristics of local entrepreneurs and companies. The structures of financial organizations, and in particular MFIs are not any different, so the network designs can benefit from considering territory requirements, activity measures as well as the risk balancing. Our manuscript extends the general territory design framework to consider risk balancing within the MFI network.

The MFI is aware that the territory centers are exposed to fluctuations in profit because of the heterogeneous nature of customers within each BU. Particularly, customers within each BU have different profits. For instance, a given BU may consist of people who knit for living and families who farm in their backyard. They would seek small size loans to cover their material expenses, and transportation expenses to go to the town and sell their material in the market. Their profits would be different from each other, creating a positive profit variance. Whereas, another BU may include farmers who may need mid or large size loans in order to purchase pesticides or rent equipment for their land. The farmers profits can also be different, potentially with a higher variance. For each BU, larger profit variance corresponds to higher risk. The MFI would not want to expose smaller establishments to the larger profit variances. For instance, a small restaurant may not be able to handle large fluctuations in customer profit as a gas station would. Therefore, MFI uses profit variance to measure risk and aims to control the risk balancing among the territory centers with respect to their tolerance using the excess profit variance.
In doing so; first, MFI estimates the profit variance for each BU using historical customer data. Then, the total profit variance, within territory centers, is used as an internal risk measure to gain insights about the customer retention rate. The purpose of the MFI is to design a micro finance network of branches and customers in which the total profit variance is adequately distributed among all branches. For instance, a branch that is located within a small grocery store, may be affected from high profit variance more than a branch that is located at a gas station. Thus, the MFI predetermines a threshold (upper limit) for the weighted total profit variance of each territory center. These weights are determined based on the type of each territory center. The threshold for each territory center is also determined by the MFI with respect to the branch type and particular region. These parameter values are provided to us as part of real-world confidential data.

Large excess amount compared to these risk thresholds constitute increased direct risk exposure for the branches, and have adverse impacts on the MFI in the mid-run. Therefore, territory centers with total profit variances that are larger than these thresholds are undesirable. Overall, customers should be allocated to territory centers so that the total excess profit variance, the difference between the weighted total profit variance and the pre-specified threshold of the territory, is minimized. We are interested in the one-sided deviation, meaning departures from the threshold value on the lower side are not constrained. This risk-balanced territory design is expected to prevent branches from having high excess profit variance. This would strengthen coordination among branches and improve customer service.

3. Territory Design Optimization. The methods of multi-criteria decision making are well studied, for instance see the reviews of Velasquez and Hester [47] and Triantaphyllou [46]. There are a variety of applications ranging from transportation [9], planning [29], economics [49], land allocation [12] to environmental policies [15]. Our focus in this paper is on multi-criteria territory design models, particularly for MFIs.

Territory design optimization models are presented in various domains such as geography, healthcare, sales and public resource management [20]. For instance, Ahmadi-Javid et al. [2] present a survey of applications in the healthcare domain, whereas Perez et al. [35] model the reconfiguration of a fire station and fleet locations for a fire department. The following overview is by no means exhaustive, see for instance Drezner and Hamacher [11] and Nickel and Puerto [34] for broader surveys on location theory and applications. Hotelling models are also relevant in which they address network facility location problems in competitive environments, see the overviews of Brenner [7] and Pinto and Parreria [36].

The main design criteria for facility location applications can be listed as geographical, activity-related and organization-based [50]. For instance, Ronen [41] minimizes the traveling distance subject to geographical constraints. Economic indicators such as number of customers, demand, workload, sales and profit allocation constitute activity-related criteria [51]. The use of activity measures for territory design has been restricted to the use of their mean or total values. Risk in territory design has mostly been attributed to the uncertainties in customer demand and costs. In their survey, Melo et al. [30] list facility location models in supply chain environment that consider risk in terms of robustness, reliability and pooling, see also Shen [43] for a discussion. Risk, as a function of activity measures, has not been considered in the territory design literature.
The models presented in literature mainly differ from each other by their distance metrics in objective functions and constraints, see the survey of ReVelle and Eiselt [37] for an overview and ReVelle et al. [38] for a bibliography. While the objectives generally include either dispersion or balancing, commonly used constraints in territory design models include balancing, connectivity, joint/disjoint assignment and similarity with the existing plan. Mixed integer programs are generally chosen for such territory design models. Rios Mercado and Fernandez [39] use a p-center model and consider the compactness and connectivity without joint assignment constraints. Salazar-Aguilar et al. [42] consider p-median and p-center models, with double balancing and connectivity constraints. They show that p-median model may be preferred rather than p-center models to handle connectivity constraints for large-scale problems. Our problem requirements are similar to these and we utilize the variation of the p-median model presented by Tansel et al. [45] to deal with balancing via constraints. Rios-Mercado and López-Pérez [40] investigate the allocation of customers to a pre-specified set of territory centers while considering assignment, balancing, similarity and connectivity constraints. None of these multi-objective decision models consider the overall territory risk and its balancing. Such consideration of risk balancing is especially important in the case of MFIs. This study aims to fill this gap in the multi-objective territory design literature by incorporating risk balancing into the decision-making framework, which is the major novelty of our manuscript. Next, we introduce the proposed optimization model in detail.

3.1. Proposed Optimization Model. Our problem definition requires making decisions on the location of territory centers as well as allocating the customers to these territory centers with respect to the activity measures, risk and constraints related to balancing, connectivity, joint assignment, and similarity with the existing plan and balanced branch type representation. The objective is to minimize the total distance among the customers (BUs) and their corresponding territory centers while favoring existing assignments from the existing plan and to minimize the total excess profit variance. Our model is one of the first territory design models within MFI literature. The allocation related constraints of our model are similar to Rios-Mercado and López-Pérez [40]; however, we also consider the location problem, as well as risk, as a function of an activity measure. In addition, we incorporate the risk balancing among branches with respect to an activity measure, as a novelty within a territory design optimization setting. Tables 1, 2 and 3 introduce the descriptions for the sets, decision variables and parameters, respectively.

<table>
<thead>
<tr>
<th>Set</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>set of all branches</td>
</tr>
<tr>
<td>V</td>
<td>set of all BUs</td>
</tr>
<tr>
<td>F</td>
<td>set of existing (former) territory centers</td>
</tr>
<tr>
<td>K</td>
<td>union set of BUs that were assigned to each territory center from set F</td>
</tr>
<tr>
<td>H</td>
<td>set of pairs of BUs that must be assigned to different territories</td>
</tr>
<tr>
<td>N_i</td>
<td>set of nodes which are adjacent to the i^{th} branch; i ∈ I</td>
</tr>
<tr>
<td>C</td>
<td>set of unconnected BUs assigned to each branch</td>
</tr>
<tr>
<td>N_C</td>
<td>union set of all BUs that are adjacent to any member of C</td>
</tr>
</tbody>
</table>

Table 1. Mathematical notation and description for sets
Decision Variable Description
\[
\begin{align*}
X_{ij} \; &\forall i \in I, j \in V \quad \text{set of all branches} \\
Y_i \; &\forall i \in I \quad \text{set of all BUs}
\end{align*}
\]

Table 2. Mathematical notation and description for decision variables

Parameter Description
\[
\begin{align*}
d_{ij} & \quad \text{Euclidean distance between nodes } i^{th} \text{ branch, } j^{th} \text{ BU; } i \in I, j \in V \\
w_1 & \quad \text{Weight of the importance of similarity with the existing design} \\
M_{ij} & \quad \text{Binary; if } j^{th} \text{ BU is assigned to } i^{th} \text{ branch in the existing plan, } i \in F \\
w_{2i} & \quad \text{Weight of the risk function for each } i^{th} \text{ branch; } i \in I \\
PV_j & \quad \text{Profit variance of } j^{th} \text{ BU; } j \in V \\
\gamma_i & \quad \text{Threshold for total profit variance of } i^{th} \text{ branch; } i \in I \\
p & \quad \text{Number of territory centers} \\
v^m_j & \quad \text{Activity measure } m \text{ for } j^{th} \text{ BU; } j \in V, m = 1, 2, 3 \\
\mu^m_i & \quad \text{Target level of activity measure } m \text{ for } i^{th} \text{ branch; } i \in I, m = 1, 2, 3 \\
t^m & \quad \text{Territorial tolerance with respect to } m^{th} \text{ activity measure; } m = 1, 2, 3 \\
\delta_i & \quad \text{Maximum travel distance for BUs assigned to the } i^{th} \text{ branch; } i \in I \\
g_{ib} & \quad \text{Binary; indicating if } i^{th} \text{ branch is of type } b \text{ or not; } b = 1, \ldots, 5 \\
l_b & \quad \text{Lower bound for the number of branches selected of type } b; b = 1, \ldots, 5 \\
u_b & \quad \text{Upper bound for the number of branches selected of type } b; b = 1, \ldots, 5 
\end{align*}
\]

Table 3. Mathematical notation and description for parameters

Activity measures; total workload, total dollar amount and total profit allocation are indexed by the values \( m = 1, 2 \text{ and } 3 \) respectively. The optimization model is given as:

\[
\min \sum_{i \in I} \sum_{j \in V} X_{ij}d_{ij} - w_1 \sum_{i \in F} \sum_{j \in K} M_{ij}X_{ij} + \sum_{i \in I} \left( w_{2i} \sum_{j \in V} (X_{ij}PV_j) - Y_i \gamma_i \right) 
\]

subject to:

\[
\sum_{i \in I} X_{ij} = 1, \quad \forall j \in V, \quad (2)
\]

\[
X_{ij} \leq Y_i, \quad \forall i \in I, j \in V; \quad (3)
\]

\[
\sum_{i \in I} Y_i = p, \quad (4)
\]

\[
Y_i \mu^m_i(1 - t^m) \leq \sum_{j \in V} X_{ij}v^m_j \leq Y_i \mu^m_i(1 + t^m), \quad \forall i \in I, m = 1, 2, 3, \quad (5)
\]

\[
X_{ij}d_{ij} \leq Y_i \delta_i, \quad \forall i \in I, j \in V, \quad (6)
\]
\[
    l_b \leq \sum_{i \in I} Y_i g_{ib} \leq u_b, \quad b = 1, \ldots, 5, \quad (7)
\]

\[
    X_{ij} + X_{ih} \leq 1, \quad \forall i \in I, \forall \{j, h\} \in H. \quad (8)
\]

The proposed model is designed with three objectives in mind. In optimization models with multiple objectives, different approaches can be used to reflect the preferences among objectives [27]. In this manuscript, we utilize the approach in which the preferences are articulated a priori. These methods are based on letting the user indicating preferences before running the optimization algorithm and allow the algorithm to come up with a solution that presumably reflects the user preferences. Particularly, we use deterministic parameters to reflect decision-maker preferences within a weighted sum model and the differences in objective-function magnitudes. In collaboration with MFI, objective function weights are determined using the maximum potential statistical range and their preferences of the importance of each term in the objective function. Since this is not our emphasis, we have not presented a comparison of other potential ways to determine weights. The reader is referred to the survey of Marler and Arora [28] for a conceptual discussion of approaches that use the weighted sum method.

The objective function is given by Equation (1). The first term refers to the total Euclidean territory distance between each BU and its corresponding territory center. The second term is a weighted penalty term that favors the assignments from the existing plan. This helps the MFI to retrieve a closer solution to the existing one, thus an implementable design. The third term corresponds to the total excess profit variance which is the total difference between the threshold and weighted total profit variance for each territory center. The weights of the risk component are determined with respect to branch type. Using the feedback of MFI and historical data, the profit variance values of BUs are assumed to be linearly independent, for simplicity. Equation (2) is the unique assignment constraint that ensures that each BU is assigned to only one branch. Equation (3) checks for the requirement that BUs are only assigned to branches that are selected as territory centers. Equation (4) ensures the construction of exactly \( p \) territory centers. Equation (5) guarantees that each territory center is within a particular tolerance level of the target level with respect to each activity measure. This helps retrieve a balanced design with respect to activity measures. The travelling distance to each territory center is not allowed to be larger than a threshold value, see Equation (6). The total number of each branch type is to be between pre-specified minimum and maximum levels via Equation (7). The pairs of BUs that need to be assigned to different territories due to organizational preferences or geographical issues (e.g., rivers or mountains) are modeled explicitly as hard constraints via Equation (8). The connectivity constraint set includes exponential number of constraints; therefore, it is impossible to present all of them explicitly. Within the solution framework, connectivity is considered for a given set of territory centers using the constraint set:

\[
    \sum_{q \in N^C} X_{iq} - \sum_{j \in C} X_{ij} = 1 - |C|, \quad \forall i \in P, C \subset \{(i, j) : X_{ij} = 1\}, N^C = \bigcup_{j \in C} N^j \quad (9)
\]

The discussion of this constraint set is provided in detail in the following section.
4. Solution Framework. The algorithms to solve territory design models can be broadly described into two groups: exact solution methods and heuristics. Exact solution algorithms such as branch and bound, branch and cut, column generation and decomposition methods are widely popular and available in commercial software. However, for large scale problems finding the exact solutions of the territory design models become challenging. Whereas heuristics based approaches may result in good feasible decisions within practically reasonable computational times. Heuristics such as Lagrangian relaxation, fixing variables, penalizing the assignment of distant pairs, perturbation, and dynamic relocation of territory centers are proposed to solve large scale territory design problems.

In our problem, the proposed optimization model results in more than 5,512,000 constraints and 5,020,000 decision variables for the real-world data instance of the MFI with 10,000 BUs and 500 territory centers. In addition, the number of connectivity constraints is exponential; thus, it is impossible to state these constraints explicitly. The computational time to solve this model to optimality is prohibitively large for the practical business user application using X-PRESS MIP Solver from FICO (Fair Isaac, formerly Dash Optimization). We had limited success even finding feasible solutions for our model within a reasonable CPU time. Therefore, at the expense of guaranteeing optimality, we use a hybrid heuristic after decomposing the problem into two levels. A similar two-level approach using different heuristics is utilized by Exposito-Izquierdo et al. [14] to provide an approximate solution to the clustered capacitated vehicle routing and allocation problem. In the first phase, we solve a relaxed location problem using a number of allocation constraints. The second phase deals with the final assignment of customers to territory centers with respect to the planning criteria and risk. In doing so, we utilize the common heuristics that are used in similar mixed integer programming based models. Using a similar set of heuristics, Rios-Mercado and López-Pérez [40] focus on the allocation phase for a given set of territories. They empirically show that heuristics provide reasonably close solutions to optimality at reasonable computational time, which make them good alternatives to solve mixed-integer programs with large scale instances. They report the relative optimality gaps in that the primal solution found under the given strategy and the best known lower bound for that instance are compared. In Lópex et al. [24], they test the utilized heuristic using an experimental design and statistical analysis in order to verify the quality of the solutions. Particularly, they compute the relative optimality gaps and investigate the interplay of the heuristic parameters, and present an analysis of the validity of the algorithm. These hybrid heuristics are shown to provide reasonable optimality gaps. We utilize a combination of these approaches to solve the model at hand, with respect to the business specifications of the MFI of concern. The solution approach is broadly based on an iterative cut generation strategy within a branch and cut framework.

The initial optimization model, $M_0$, is a relaxed p-median model that determines the location and allocation decisions as warm start. It only considers the unique assignment and capacity constraints in a reduced set. This set $R$ corresponds to a reduced subset in which $X_{ij}$ is set to be 0 for all distant BUs and branches $\{(i,j) \in R\}$. A heuristic parameter, KERNEL, is defined to determine the coverage of the set $R$. We assume that assigning BUs to the far away branches is unlikely in practice [40], therefore we do not consider those potential assignments. The MFI simply would not want the assignment of customers that are far away from the branches, since that would make their interaction difficult.
In addition, we relax the consideration of activity measure constraints by employing a user-chosen $H_1$ heuristic value. The updated constraint is written as:

$$\sum_{j \in V} X_{ij} v_j^m \leq H_1 Y_i \mu_i^m (1 + t_m), \quad \forall \{(i, j)\} \in R, \forall i \in I, m = 1, 2, 3 \quad (10)$$

This heuristic gives the flexibility to consider a larger set of potential assignments with respect to activity measures. Rios-Mercado and López-Pérez [40] investigate this tradeoff and provide computational analysis. In our setting, after initial experimenting, we decided to set $H_1$ value as 5. The optimization model, $M_0$ is given as:

$$\min \sum_{i \in I} \sum_{j \in V} X_{ij} d_{ij} - w_1 \sum_{i \in I} \sum_{j \in K} M_{ij} X_{ij} + \sum_{i \in I} \left( w_{2i} \sum_{j \in V} (X_{ij} PV_j) - Y_i \gamma_i \right)$$

subject to:

$$\sum_{i \in I} X_{ij} = 1, \quad \forall j \in V, \{(i, j)\} \in R,$$

$$X_{ij} \leq Y_i, \quad \forall i \in I, \forall j \in V, \{(i, j)\} \in R,$$

$$\sum_{i \in I} Y_i = p,$$

$$Y_i \mu_i^m (1 + t_m) \leq \sum_{j \in V} X_{ij} v_j^m \quad \forall i \in I, m = 1, 2, 3,$$

$$\sum_{j \in V} X_{ij} v_j^m \leq H_1 Y_i \mu_i^m (1 + t_m), \quad \forall \{(i, j)\} \in R, \forall i \in I, m = 1, 2, 3$$

The solution of this model provides location and allocation decisions that can be used as warm-start for solving the complete model. However, the exponential number of connectivity constraints still prevents us to find reasonably good feasible solutions for the complete model. Therefore, the second phase starts with solving a relaxed model without the connectivity constraints. Then, we iteratively check if the obtained solutions satisfy the connectivity constraints, and add cuts for the violated connectivity constraints. The utilized connectivity constraints are initially proposed by Drexl and Haase [10] in the context of routing problems, and are adapted to territory optimization settings [40]. Specifically, for a set of given territory centers and using the definitions that are provided in Tables 1-3, we propose the constraint set:

$$\sum_{q \in N^C} X_{iq} - \sum_{j \in C} X_{ij} = 1 - |C|, \quad \forall i \in I, C \subset \{(i, j) : X_{ij} = 1\}, N^C = \bigcup_{j \in C} N^j$$

This constraint set is used to evaluate if the subset $C$ contains a partition of BUs that are not connected to the territory center but are still assigned to that territory. The cardinality of subset $C$ ranges from 1 up to $K/2$, where $K$ is the number of BUs assigned to the territory. If a particular BU is assigned to a territory center, there should be at least one of the neighbors of BU (i.e. $q \in N^C$) that needs to be assigned to the same territory so that the connectivity is guaranteed. Figure 2 demonstrates the impact of added cuts that decrease the disconnected BUs over time.
In addition, we utilize the algorithmic strategies such as perturbation and dynamic assignment of territory centers. The utilization of heuristics to improve solution methods for mixed integer programs has been an active area of research, for instance see the hybrid genetic algorithm of Herda and Haviar [19]. Particularly, the implementation of perturbation heuristics is shown to provide computational advantages in the solution of mixed integer programs with large instances. For an illustration, see Hane et al. [17] which presents an application in the context of fleet assignment problems. The basic idea is to add a penalty term to the objective function that would favor keeping the assignments of BUs to close territories. The smaller the distance between the jth BU and the ith territory center is, the larger the penalty will be within the objective function in order to avoid any assignment changes for the decision variable. The weight of this perturbation penalty is defined as PERT. Perturbation can adversely affect the number of interior-point iterations required. In some cases, this increase in the number of interior iterations is expected to be offset by finding a good basis quickly. A value of zero for the parameter PERT would mean that perturbation heuristic is not used at all.

Furthermore, we also utilize a heuristic that allows dynamic relocation of branches. The heuristic parameter CORE is defined as the proportion of branches to be used as candidates for territory recentering. This heuristic lets us update the assignment of territory centers in a dynamic fashion. In the case of a territory center change, the distances to the BUs that are served by that branch are recalculated and the relevant constraints are checked again. A large value of the parameter CORE results in a larger number of branches to be reconsidered as territory centers. For instance,
for the implementation with heuristic values $H_1=5$, PERT=50, CORE=0.90, Figure 3 shows the number of recentered territory centers over time. It can be seen that there is a high number of recenterings in the initial iterations, and the number of recenterings and therefore total distance stabilizes after a certain number of iterations. The rate of change in the number of recenterings decreases over time.

The investigation of the trade-off between the solution quality and computational time is beyond the scope of this manuscript, please see López et al. [24] for a similar relevant analysis of the solution framework.

5. **Analysis.** This section analyzes the impact of the proposed territory design on the MFI. In order to assess the impact of risk balancing of the territory design, we solve two versions of the proposed model. We refer to the proposed model as Model 1, and define Model 2 as the version of the proposed model without the excess profit variance (risk) component in the objective function. Model 2 is constructed to replicate the territory design using the existing variables of interest that are retrieved from the subject matter experts, but without risk. These models are compared using realistic data instances that have increasing variability. In doing so, a percentage of the basic units (so-called variability factor) are randomly chosen from the initial 10,000 BUs, and their planning criteria levels are multiplied by a factor for increased variability. The common term of the objective functions of both models is named Distance, whereas the excess profit variance term is referred to as Risk.

The upper plot of Figure 4 presents the distance values of Model 1 (bold line) and Model 2 (dashed line). In terms of “Distance”, Model 1 is always found to
result in worse solutions for distance compared to Model 2. This is mathematically expected and it can be concluded that risk balanced territory designs have worse total distances. Moreover, increased variability within data instances result in worse total distances. Whereas, the lower plot of Figure 4 presents the “Risk” values of only Model 1, since Model 2 has no “Risk” term in its objective function. It can be argued that the overall risk increases with increasing variability.

As a result of this study, we provided the MFI with the territory center locations and the allocation of respective BUs to be served by that territory. Figure 5 presents a partial map of the proposed territory design model. Each territory center is shown as a circle in the numbered territory. The relative balance of the territories can be recognized in terms of the areas they serve as well as their compactness.

Overall, the risk-balanced territory design results with an increase in the total distance travelled by customers compared to the case without risk consideration. Despite that, the MFI of interest has implemented the model since they find the consideration of risk in their territory design to be very important. A risk balanced territory design helps balance the financial risk exposure on branches within the MFI network. None of the branches are totally disadvantaged in terms of profit fluctuation. This improves the overall financial health of the micro finance network, emphasizing the importance of cooperation rather than competition among branches. Such cooperation and balanced designs is expected to result in reduced risk and a number of potential major positive outcomes. Firstly, the customer satisfaction is expected to increase as a result of such territory design. If such risk is not considered, unbalanced number of customers may increase the financial exposure and volume of a given branch. This may lead to increased churn rate of customers. Second, the branches with smaller profits and/or higher profit variance could have eventually been eliminated from the network; because of the unsustainable nature

\[ \text{Figure 4. Objective Function Values of Risk and Distance for Model 1 (bold line) and Model 2 (dashed line)} \]
of the operations due to fixed costs and low volume. This potential closure of existing branches would decrease the accessibility, effectiveness and power of the micro finance network. Closing branches and consolidations increase costs in terms of real estate, lease contracts, and human resources management.

6. Conclusion. This paper is the result of a contract between the authors and a MFI in Mexico and presents one of the first attempts of a risk-balanced territory design for micro finance institutions. This problem requires decisions on the location of the territory centers from the available branches and allocation of the customers to the territory centers with respect to risk and planning criteria; namely, the total workload, loan amounts and profit allocation. These business requirements of the MFI are modeled using a mixed integer program with a p-median dispersion objective function and constraints that satisfy balancing, connectivity, assignment, similarity with the existing plan and balanced representation of different type of branches. As a novelty for the territory design optimization models, the risk balancing of the proposed design is considered by using the total excess variance of the profit allocation for territory centers. Although optimality cannot be guaranteed, the decisions of territory locations and customer allocations are found to be beneficial and verified by the MFI, and the model is currently being implemented in a pilot-region. The modeling framework is applicable to other MFIs, in general. However, further data from MFIs in other countries should be collected to extend our study, and validate the proposed model.

The proposed framework assumes deterministic values for the parameters of the model. This limitation can be relaxed by optimizing the expected objective function.
value with respect to a number of scenarios. The study of uncertainty of profit variances and activity measures can result in more realistic and timely results at the expense of computational complexity. The large scale of the problem prevents us from studying the connectivity constraints explicitly and requires model relaxations to be able to retrieve feasible solutions. An alternative might be to run the model on pre-specified unconnected regions. This leads to the chance of retrieving optimal results for smaller scale regions at the expense of losing connectivity among regions.

There are a number of potential extensions that can be considered for the proposed model. First, another possible risk balancing function can be utilized, such as using a target value rather than a threshold to consider the differences from the total profit variance. Such approach may result in solutions with more uniformly distributed risk among branches at the expense of some branches potentially carrying a larger risk than the target. In our setting, the MFI requested the avoidance of branches with larger risk, therefore we use a threshold-based risk function. In addition, financial risk due to the loan delinquency and default can also be included within the integrated territory design. In this manuscript, loans are assumed to be given. The decision maker can also consider the impact of location of branches on profit allocation while making loan approval decisions. For instance, if there are two loan applicants with high risk in a particular territory, both may not be given loans to decrease the overall risk. In areas where there is more than one MFI, a competitive facility location model based on variants of Hotelling networks can also be considered. For instance, a Southeast Asia region, where there is competition among MFIs, can benefit from considering such an alternative and this would be a valuable extension of the current model.

The incorporation of risk balancing into the optimization framework may be applicable for similar territory design problems. For instance, a risk-balanced location-allocation model can be appropriate for the vehicle routing problem in which you decide on the number of vehicles in a given fleet first, and then allocate the customers to vehicles. In such cases, you would not want a given vehicle to be exposed to customers with more volatility. That may result in the drivers being exposed to higher risk and losing money in the short term, and eventually switching to another company.

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